MG TECHNIQUES FOR STAGGERED DIFFERENCES

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SUMMARY

The numerical solution of quasi-linear elliptic partial differential equations on general two dimensional domains is considered. The equations are approximated on a staggered grid and solved with a Multigrid technique. Several aspects of the MG method for the staggered finite difference equations are considered, with particular attention to the enforcement of Dirichlet boundary conditions and the restriction operator introduced for transfering information from fine to coarse grids. This paper aims at assessing the accuracy and the efficiency of appropriate MG techniques for staggered differences.

INTRODUCTION

As the numerical models of the heat transfer and fluid motion phenomena have become more sophisticated, treatment of variables location has been improved in order to better represent real physical processes and to construct accurate and stabile difference approximations. In particular two different discretization schemes have been proposed: the MAC scheme for the solution of Navier-Stokes eqs. [1] and the control volume energy balance approach [2]. In the MAC scheme the velocity components are defined at the cell midsides, while the pressure is displaced to the center of the cell. With this scheme major difficulties connected to the enforcement of the incompressibility constraint are overcomed and the results do not exhibit any spurious numerical oscillations. Analogously in the control volume scheme, developed for heat conduction problems, the temperature is defined at the center of the cell. In these schemes the variables location is motivated by an integral conservation law to be satisfied exactly for each individual cell. Moreover the difference approximation obtained with the MAC scheme to an elliptic differential operator have a good discrete ellipticity measure [3]. Both staggered difference schemes have been extended to generalized non-or-

orthogonal coordinates [2, 4].

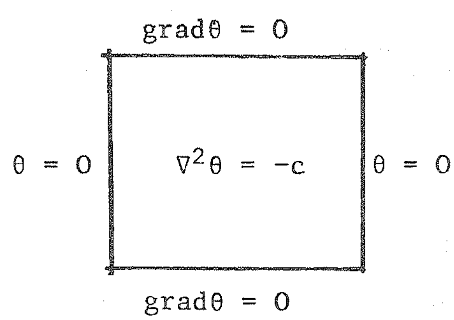
The implementation of a MG solver of staggered finite difference problems should incorporate the development of specialized relaxation techniques, appropriate transfer operators and boundary conditions representations. Up to now the attention has been focused primarly on the relaxation schemes (Convective Successive Line Relaxation [5], Distributive Gauss-Seidel [3]). Comparatively less effort has been directed to the analysis of different transfer operator on staggered grids, in particular for curvilinear meshes [5].

A MG solver as been elaborated for the solution of problems defined on complex geometries.The curvilinear coordinates system is computed using an elliptic grid generation method solved by an MG technique [6].

The first test case considered is a problem of heat conduction on a cartesian grid. Boundary conditions are selected in such a way that the analytical solution is function of one independent variable.

PRELIMINARY NUMERICAL RESULTS

The numerical technique has been applied to the heat conduction problem.



The one-dimensional analytical solution has the following expression

$$\theta = c/2\, x\, (1 - x)$$

First a linear interpolation both for the restriction operator and the boundary conditions has been considered exhibiting a truncation error of the first order. Nethertheless the results appear quite anomalous (Tab. 1): better accuracy is obtained on the intermediate grid rather than on the finest level. This behaviour may be interpreted examining the related effects of the discretization of the boundary conditions and of the transfer of the solution. The error induced by the restriction operator appears in the right hand side of the FAS correction equations. It acts like an additional source term compensating the effect of the linear interpolation of the boundary conditions. Better results are obtained on the finest level if the boundary conditions are represented by a second order plynomial. Indeed on the finest grid only the correction is transferred, which in the present problem is not affected by the interpolation error of the solution. Instead on the coarser levels this error spoils the accuracy of the solution. Therefore a higher order restriction operator should be employed to improve the accuracy on each grids.

CASE / LEVEL	ERRORS L2 - NORM A	B	C
4 x 4	.31-02	.39-02	.22-06
8 x 8	.16-06	.48-03	.15-06
16 x 16 (h = .0625)	.27-03	.98-07	.83-07

Tab. 1: A) Linear interpolation of boundary conditions and restriction.
B) Second order b.c. and first order restriction operator.
C) Second order b.c. and fourth order restriction operator [7].

CONCLUDING REMARKS

Presently the work is in progress on more elaborate test-cases for two dimensional problems on cartesian grids. The aim is to test the general validity of the conclusion obtained through the analysis of the present problems, which is essentially one-dimensional.

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