LOCAL REFINEMENT
ALGORITHMS

D. BAI

Local refinement algorithms

I) Application of local refinement algorithms

1) Solution of inverse problems, like the gravimetric problem, where large accuracy is needed in some parts of the domain, e.g. near surface electrodes.

2) Solution of differential equations with different types of singularities.
When there is a large truncation error near the singularity, a large solution error far from the singularity results.
The solution error can be efficiently reduced by local refinement algorithms.

II) Euler equation for minimizing solution error for a given ammount of work is:

$$G \frac{\partial \tau}{\partial h} - \lambda d\, W(p)\, h^{-d-1} = 0$$

where:

$G(x)$ - error weighting function.

$\tau(x)$ - local truncation error of the differential equation.

d - dimension

$W(p)$ - the work per grid point. p is the order of the equation.

λ - marginal rate of exchanging optimal accuracy for work ($\lambda = -\frac{dE}{dW}$)

III) Numerical experiments

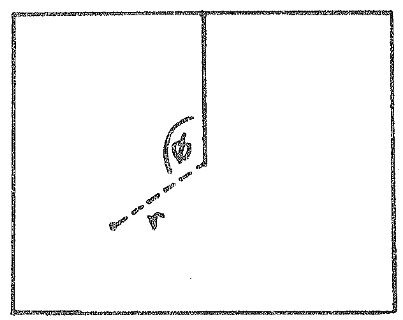
The asymptotic convergence rate is the same as in regular problems. In structural singularities (not RHS or BC) this can be achieved by using additional relaxations on very small parts of the domain only, so that the additional work needed is very small.

1) Singularity in right hand side of equation. For example: $\Delta u = F$, with the solution $u = r^{\alpha}$. In this case Euler equation gives **WE= Const.** as for the regular case!

In actual numerical experiments, for this problem, there are slight changes from WE = const. because there are changes in the sign of the truncation error in the domain.

2) Singularities resulting from the shape of the boundary (structural singularities).

For example, the domain



with the solution $u = r^{1/2} \sin \frac{\phi}{2}$.

of Laplace's equation $\Delta u = F$. In this case also, Euler equation gives

WE = Const. as for the regular case!

IV) Algorithmic flow problems:

1) Regular FMG is not sufficient for local refinement because too much work is invested when adding a grid covering only small region of the domain. This can be improved by using λ-algorithm in which λ is decreased in a gradual sequence

2) Interpolation to the boundary of a fine grid, covering only part of the domain: The interpolation to fine grid boundary points which are not common to both the coarse and fine grids should be of order $p+m$ where p-is the order of approximation, and m-is the order of the equation.

V) Current research:

Sources singularities with some surprising results