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**Numerical Method for Dynamic Bertrand Oligopoly
Using Multigrid**

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In economics, oligopoly is a market structure in which a small number of firms producing substitutable goods compete by setting prices strategically in an uncertain demand environment (Bertrand model). For instance, Pepsi and Coca-Cola in the soft drink industry, Apple and Samsung in the smart phone industry. Firms choose prices to maximize profit in the sense of Nash Equilibrium. The Bertrand Oligopoly can be modelled as nonzero sum differential games. The model can be solved by the dynamic programming principle, which yields a system of Hamilton-Jacobi-Bellman (HJB) equations. More precisely, let V_i be the value functions, which represent the expected discount lifetime profit. Then for the special case where there are only two players (duopoly), the unknowns V_i satisfy:

$$\frac{\partial V_i}{\partial t} = LV_i - D_M(p_j) \frac{\partial V_i}{\partial x_j} + \sup \left\{ D_i(p_i, p_j) p_i - \left(\frac{\partial V_i}{\partial x_i} - \frac{\gamma}{\beta} \frac{\partial V_i}{\partial x_j} \right) \right\} - rV_i,$$

where p_i is the price for goods produced by firm i , and L is the differential operator:

$$L = \frac{1}{2} \sigma_1^2 \frac{\partial^2}{\partial x_1^2} + \rho \sigma_1 \sigma_2 \frac{\partial^2}{\partial x_1 \partial x_2} + \frac{1}{2} \sigma_2^2 \frac{\partial^2}{\partial x_2^2}.$$

In this talk, we will present a numerical method for solving the system of HJB PDEs. We will discuss the discretization of the nonlinear equation, the issues of viscosity solutions, and present a monotone finite difference scheme. We will then solve the resulting discrete systems by a multigrid method. We will present a relaxation scheme as a smoother, and a prolongation method for the system case. Finally, numerical results will be presented to illustrate the effectiveness of the method.