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**Multigrid Method for Solving Elliptic Monge-Ampere
Equations Arising from Image Registration**

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The Monge-Ampère equation (MAE) is a nonlinear second order partial differential equation, which can be written as:

$$u_{xx}u_{yy} - u_{xy}^2 = f, \quad \text{on } \Omega \quad (1)$$

where $f > 0$ so that the equation is elliptic with a unique convex solution. Monge-Ampère equations arise in many areas such as differential geometry and other applications. In image registration, one is interested in transforming one image to align with another image. One approach is based on the Monge-Kantorovich mass transfer problem. The goal is to find the optimal mapping M which minimizes the Kantorovich-Wasserstein distance. The optimal mapping can be written as $M = \nabla\psi$, where ψ satisfies the following Monge-Ampère equation

$$\det(D^2\psi(x)) = \frac{I_1(x)}{I_2(\nabla\psi)},$$

where I_1 and I_2 are the given images. Here $\det(D^2\psi(x))$ denotes the determinant of the Hessian of ψ . In $\mathbb{R}^2$, it is equivalent to (1). In this talk, we will present a multigrid method for solving the Monge-Ampère equation. We will discuss the discretization of the nonlinear equation and the issues of viscosity solutions and monotone finite difference and finite element schemes. We will then present a relaxation scheme which is a very slow convergent method as a standalone solver but it is very effective for reducing high frequency errors. We will adopt it as a smoother for multigrid and demonstrate its smoothing properties. Finally, numerical results will be presented to illustrate the effectiveness of the method.