Christopher Siefert

Multilevel Preconditioners for the XFEM with Weak Discontinuities

Computational Shock and Multiphysics Department
Sandia National Laboratories
P O Box 5800
MS 1323
Albuquerque
NM 87185-1323
csiefer@sandia.gov
Richard Kramer
Thomas Voth

In multimaterial problems, surface and interface effects have the potential to be of great importance, for instance in electromagnetics, where current tends to concentrate on material surfaces, In cases where Lagrangian (moving) meshes are untenable due to large-scale deformation, we are often left with the alternative of using an Eulerian (non-moving) meshes coupled with some sort of mixture model. For some types of physics, these mixture models are quite good, in others they leave much to be desired.

The extended finite element method (XFEM) has primarily attracted attention in the computational mechanics for its ability to handle strong discontinuities in the solution field such as cracks. However, the XFEM has also been shown to be applicable to weak discontinuities as well, such as the material interface problems discussed above. Unfortunately, the state of multilevel preconditioners for the XFEM is somewhat behind the state of the discretizations themselves, as many of these discretizations are presented without a solver to accompany them.

We present a simplified XFEM employing pointwise constraints to enforce C^0 continuity, which displays almost identical convergence behavior to the method of vital vertices [1]. We show two multilevel preconditioners for this method. The first is an auxiliary preconditioner based on a FEM discretization without the material interface. The second method is based on a change of basis to an equivalent body-fit FEM problem. Results for both multilevel preconditioners are presented for model problems in two dimensions.

[1] M. Hautefeuille, C. Annavarapu, and J. Dolbow. Robust imposition of Dirichlet boundary conditions on embedded surfaces, IJNME, in press, 2012.