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**Eigenvectors of Matrices Associated with Scale-Free
Graphs**

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A scale-free graph is characterized as having vertices with a degree distribution that follows a power law. Clustering, ranking, and measuring similarity of vertices on scale-free graphs are all quantities that network scientists often calculate. We describe a few network calculations that are obtained by solving linear systems, eigensystems, nonlinear constrained optimization problems, and tensor decompositions involving matrices (or tensors) whose sparsity structure is related to an underlying scale-free graph. In each of these settings, linear solves are often fundamental calculations performed as subroutines within the larger iterative solution process. For many large systems of interest, classical iterative solvers (such as conjugate gradient) converge with prohibitively slow rates, due to ill-conditioned matrices and small spectral gap ratios. Scalable preconditioners for this class of problem are of considerable research interest.

We consider constructing algebraic multigrid (AMG) preconditioners, which make use of coarse approximation of eigenspaces to accelerate convergence. We discuss several interesting properties the eigenvectors of scale-free graphs have that are not present in mesh-like graphs. For example, these graphs often have a large number of locally supported eigenvectors (LSEVs), eigenvectors that are each nonzero only in a small, connected region. Another common related property is that eigenvectors in a matrix's near-nullspace are essentially local, having significant magnitude in a very small region of the graph and rapidly decaying away from this region. We discuss how such properties can be used to coarsen graphs with potentially high spectral accuracy and ultimately improve the scalability of preconditioned iterative methods.

Additionally, these properties are important for understanding how the quality of a numerical solution relates to the quality of the data mining task, loosely analogous to the relationship between discretization error and optimal convergence tolerances in numerical solutions to partial differential equations.