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An efficient multigrid algorithm for finite difference discretizations on triangular grids

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The differential operators divergence, gradient and rotor are often used to formulate mathematical physics problems. To approximate the solution of this type of problems it is convenient to use irregular grids to fit better the spatial domain. A natural way to discretize the differential problem is to define the corresponding discrete grid operators. These must satisfy the main properties of the continuous operators and also some compatibility relations between them. In the literature the associated methods are called mimetic finite difference methods. In a recent paper [1], a specially clear approach, called VAGO (Vector Analysis Grid Operators) method, has been proposed on Delaunay triangular grids. For this discretization, it is not necessary to use a concrete coordinate system. This fact is specially interesting when computational irregular grids are considered.

An important issue in solving this type of problems is the efficient solution of the corresponding algebraic system of equations which results after the discretization process, and multigrid methods are generally accepted as the fastest numerical methods for the solution of elliptic partial differential equations. Although the algebraic multigrid is a useful tool to solve problems on unstructured grids, we consider in this work an efficient and robust geometric multigrid algorithm to solve problems on triangular grids in a free-matrix version. This is achieved by using an adequate data structure resulting in a very fast solver with very low memory requirements. Finally, we consider some test problems to show the goodness of this multigrid method.

[1] P.N. Vabishchevich, Finite difference approximation of mathematical physics problems on irregular grids, Comp. Meth. Appl. Math. 5 (2005) pp. 294–330.