
Stephanie Friedhoff
**A multigrid-in-time algorithm for solving evolution
equations in parallel**

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We consider optimal-scaling multigrid solvers for the linear systems that arise from the discretization of problems with evolutionary behavior. Typically, solution algorithms for evolution equations are based on a time-marching approach, meaning solving for one time step after the other. These traditional time integration techniques lead to optimal-scaling but not parallelizable algorithms. However, current trends in computer architectures are leading towards more, but slower, processors and, therefore, driving the need for greater parallelism. One approach to achieve parallelism in time is with multigrid, but while classical multigrid methods rely on multiscale representations in space, that arise naturally from decomposing a function into a hierarchy of frequencies from global smooth modes to local oscillations, these approaches do not extend to evolutionary variables in a straightforward manner, because of the fundamentally local structure of the evolution. In this talk, we present an optimal and scalable multigrid-in-time algorithm for diffusion equations as simple examples of evolution equations. Our algorithm is based on interpreting the parareal time integration method [J. L. Lions, Y. Maday, and G. Turinici, *Résolution d'EDP par un schéma en temps pararéel*, C.R. Acad. Sci. Paris, Sér. I Math, 332 (2001), pp. 661-668] as a two-level reduction scheme, and developing a multilevel algorithm from this viewpoint. We demonstrate optimality of our algorithm for solving the one-dimensional diffusion equation in numerical experiments. Furthermore, by using parallel performance models, we show that we can expect speedup in comparison to sequential time-marching on modern architectures.