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**Adaptive Algebraic Multigrid for Singular Value
Decomposition**

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An adaptive (or ‘self-learning’) algebraic multigrid (AMG) method is described for computing a few of the largest or smallest singular values of a rectangular matrix, and their associated singular vectors. The proposed algorithm combines and extends two existing AMG approaches for symmetric positive definite eigenvalue problems. The method consists of two multilevel phases. In the first, multiplicative phase (setup phase), tentative singular triplets are calculated along with a multigrid hierarchy of interpolation operators that approximately fit the tentative singular vectors in a collective and self-learning manner, using multiplicative update formulas and bootstrap AMG interpolation. In the second, additive phase (solve phase), the tentative singular triplets are improved up to the desired accuracy by using an additive correction scheme with fixed interpolation operators, combined with a Ritz update. A suitable generalization of the singular value decomposition is formulated that applies to the coarse levels of the multilevel cycles. The combined multiplicative-additive approach results in an AMG method for extremal singular triplets that combines two desirable properties: it allows for high-accuracy convergence when desired, and it is flexible enough to deal efficiently with a variety of problems due to its self-learning properties. Numerical tests show that the algorithm converges to high accuracy in a modest number of iterations for singular value and eigenvalue problems from different application areas. Extension of the approach to canonical tensor decomposition is also briefly discussed.