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Second Order Multigrid Methods for Elliptic Problems with Discontinuous Coefficients on an Arbitrary Interface

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Elliptic equations with discontinuous coefficients across a codimensional-one interface arise in several applications. Interface-fitted grid methods are difficult to use in the case of moving interfaces, because of the computationally expensive meshing procedures needed at each time step. In such cases an approach treating the interface as embedded in a Cartesian grid may be preferred.

Multigrid technique is one of the most efficient strategy to solve a class of partial differential equations, using a hierarchy of discretizations. Several multigrid approaches exist in literature to treat the jumping coefficient problem in 2D when the interface is aligned with the Cartesian grid. We mention the method based on operator-dependent interpolation (Brandt et al., Dendy et al., ...), where the interpolation is carried out by exploiting the continuity of the flux instead of the gradient of the solution, and the method based on Galerkin Coarse Grid Operator (Stuben et al.).

In this work, we propose a multigrid technique to solve the Poisson equation with discontinuous coefficients along an arbitrary interface, described by a level-set function in a Cartesian grid. A proper treatment of the transfer operators and the relaxation on the boundary is proposed, in order to achieve the best convergence factor (i.e. the convergence factor predicted by the Local Fourier Analysis for the inner relaxation and in a rectangular domain). The convergence factor does not depend on the size of the grid, nor on the jump in the coefficient. Therefore, this solver can be employed for instance in a fluid dynamic context, where a two-phase flow is modeled by the incompressible Navier-Stokes equations and the jump in the coefficient consists of the jump in the viscosity or the density between the two fluids.

The method is second order accurate in the solution and the gradient.