## Max la Cour Christensen Nonlinear Multigrid for Reservoir Simulation

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In the pursuit of higher resolution simulation models that use all seismic, geological, and dynamic reservoir data - and to make use of modern parallel computing architectures - we consider alternative numerical methods to solve the system of equations governing subsurface porous media flow.

The subject of this work is a thorough investigation of the application of nonlinear multigrid techniques, specifically the Full Approximation Scheme (FAS), for simulation of subsurface multiphase porous media flow. The main motivation for addressing this topic is a need for higher resolution and efficient simulations leading to better decision making in the production of oil and gas. Higher resolution simulations require efficient utilization of many-core parallel architectures. Current numerical methods employed in industrial reservoir simulators are memory intensive and not readily scalable on large-scale distributed systems and modern many-core architectures such as GPUs or Intel MICs.

In a first step, we investigate alternative numerical methods to establish algorithmic performance in serial computations. The nonlinear multigrid technique FAS uses local linearization, which allows for local components suitable for parallel implementation. Furthermore, FAS is a memory lean algorithm. By using FAS, we avoid having to assemble the Jacobian on the finest grid, which results in major memory savings. To our knowledge, very little work is published on FAS for reservoir simulation. Previous work by Molenaar considers the application of FAS on a simple 2D immiscible two-phase no gravity homogeneous example. To our knowledge, FAS has not been applied successfully to more complicated heterogeneous reservoir problems.

Two reservoir simulators have been implemented in C++ in serial. The first simulator is based on conventional techniques with global linearization in Newtons method and state-of-the-art choice of methods for the linear solver. Specifically, we use the Krylov subspace method FGMRES preconditioned with the Constrained Pressure Residual (CPR) method. CPR is a two-stage preconditioner based on a decomposition of the full system into a pressure system, for which

the solution is then used to correct the full system. The intermediate pressure system is solved by an algebraic multigrid method. The second stage of CPR is done by applying  $\mathrm{ILU}(0)$  to the corrected full system.

The second simulator is based on the nonlinear multigrid method FAS. Both simulators solve the same system of PDEs governing 3D three-phase flow of oil, water and gas in a subsurface porous medium taking into account gravitational effects. The same discretization techniques are used for both simulators. For spatial discretization, the Finite Volume Method is used and for temporal integration, the backward Euler method is used. This enables fair comparisons between the conventional methods and FAS.

The two reservoir simulators have been tested extensively to compare the nonlinear multigrid approach FAS with the conventional techniques applied in modern reservoir simulation. The test models range from homogeneous to highly heterogeneous models. It has been demonstrated that, without loss of robustness, FAS outperforms the conventional techniques in terms of algorithmic and numerical efficiency for the model equations considered. Furthermore, memory comparisons have been carried out, which show that FAS provides a significant memory reduction in comparison with conventional techniques. This memory reduction is an attractive feature, which enables higher resolution simulation for the before mentioned modern many-core architectures.