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Local Fourier Analysis for Block Smoothers in Multigrid Methods

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On parallel computers and nowadays computer architectures block smoothers possess the advantage of a high computation/communication and/or a computation/memory access ratio, especially when compared to traditional point smoothers. Therefore, multigrid methods should benefit from the use of block smoothers regarding time to solution.

While block smoothers like block Jacobi are used in current parallel implementations of multigrid methods, they have not been studied extensively. We extended the traditional Local Fourier Analysis described in the literature in detail e.g. in the book of Wienands and Joppich for the case of block smoothers. Unlike the previous study of overlapping smoothers in the recent paper by MacLachlan and Oosterlee (Num. Lin. Alg. Appl., 18) our approach is based on a decomposition of the infinite Toeplitz matrices and considers matrix-valued generating symbols instead of scalar generating symbols.

We present our approach and provide numerical examples for different block smoothers, including smoothers with and without overlap and different coarsening ratios and block sizes. Additionally, we consider different solution strategies for the block solves, including direct solvers as well as iterative methods.