
James Adler
**Constrained First-Order System Least Squares for
Improved Mass Conservation**

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In complex fluid flow simulations, there is a tradeoff between obtaining solutions that are accurate with a reasonable amount of computational work and satisfying certain conservation laws exactly. For instance, in incompressible fluid flow, conservation of mass takes the form of making sure the fluid velocities are divergence-free. In magnetohydrodynamics, one must satisfy conservation of mass as well as the solenoidal constraint that the magnetic field is divergence-free (i.e. there are no magnetic monopoles). Many methods have been applied to such systems, some being conservative at the cost of accuracy of the momentum equations and others at the cost of efficiency in the solver. First-order system least-squares approaches have also been applied and yield efficient methods for approximating solutions to coupled fluid mechanics problems. However, without proper care, the auxiliary conservation equations may not be solved to a sufficient accuracy. In this talk, we propose a constrained least-squares approach, where we augment the first-order system and minimize the least-squares functional subject to some constraint. Here, we only look at a simple diffusion equation, but present the main ideas, including what types of finite-element spaces to use and the solution algorithm. A domain decomposition or multilevel approach is employed to solve the constrained problem on local subdomains and coarse grids and used to update the unconstrained solution as needed. Thus, we approximate the solution accurately and efficiently using the least-squares method, while still conserving the appropriate quantity.