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On Optimized Schwarz Preconditioning for High-Order Spectral Element Methods

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Optimized Schwarz preconditioning is applied to a spectral element method for the modified Helmholtz equation and pseudo-Laplacian arising in incompressible flow solvers. The preconditioning is performed on an element-by-element basis. The method enables one to use non-overlapping elements, yielding an effective algorithm in terms of communication between elements and implementation. Two approaches are tested. The first consists of constructing a P_1 finite element problem on each overlapping element. In the second, the preconditioner is applied directly on a non-overlapping spectral element. Numerical results demonstrate an improvement in the iteration count over the classical Schwarz algorithm.

Introduction

The classical Schwarz algorithm uses Dirichlet transmission conditions between subdomains. By introducing a more general Robin boundary condition, it is possible to optimize the convergence characteristics of the original algorithm (Charton et al. 1991; Chevalier et al. 1998; Gander et al 2002; Gander 2003). In this work, a study of the model equations $u - \Delta u = f$ and pseudo-Laplacian arising in incompressible flow solvers is performed. As suggested by the work of Fischer et al. (2000), the preconditioning is either implemented via a P_1 finite element formulation of the original problem build on the spectral element grid, or directly by solving a smaller spectral element problem without overlap on each spectral element. Although traditional Schwarz preconditioning combined with a coarse grid solver is quite efficient, the need for even more powerful preconditioning techniques stems from atmospheric modeling. Recently (see Thomas and Loft 2002; St-Cyr and Thomas 2004), a semi-implicit SEM was combined with OIFS time stepping (Maday et al. 1990), enabling time steps on the order of 20 times the advective CFL condition (Xiu and Karniadakis 2001). This directly reflects as a significant increase in the number of conjugate gradient iterations required to perform the semi-implicit step.

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