In this talk we present a class of algorithms for the parallel numerical solution of nonlinear partial differential equations arising in both simulation and optimization problems. The algorithm can be considered as an improved version of inexact Newton methods through the use of nonlinear Schwarz preconditioning, adaptive step-length for line search, and adaptive merit functions. In addition, we couple the Krylov stopping conditions with the quality of the Newton direction with respect to the merit functions. Because of the flexibility on the overlapping size, the type of restrictions, the number of levels, and the inexactness of the subdomain solvers, Schwarz preconditioners constitute a key element for the overall robustness and scalability of our proposed algorithm. We report computational results for some two-dimensional flows described by incompressible Navier-Stokes equations with different combinations of Reynolds numbers, grid sizes and number of processors.