Emeric MARTIN

Using spectral low rank preconditioners for large electromagnetic calculations.

CERFACS
42 Avenue Gaspard Coriolis
31057 Toulouse Cedex
France.
emartin@cerfacs.fr
Iain DUFF
Luc GIRAUD
Julien LANGOU

We focus on the solution of a sequence of linear systems arising in electromagnetic radar cross section, and having the same coefficient matrix but different right-hand sides. The problem consists in solving $M_1AX = M_1B$, where M_1 is a left preconditioner, A a large dense complex symmetric matrix that arises from boundary element method, X the block of unknowns vectors, and B the block of right-hand sides.

Our study starts from the observation that when the matrix M_1A has some eigenvalues near zero, the convergence of the Krylov methods is often slow. The following proposition from [1] shows that we can construct an update $\tilde{M}_c$ from spectral information of M_1A to correct M_1 such as the new preconditioned system $M_2Au = M_2b$ no longer has eigenvalues in a certain neighbourhood of zero. Assume that M_1A is diagonalizable:

$$M_1A = V\Lambda V^{-1},$$

with Λ the diagonal matrix formed by the eigenvalues $\{\lambda_i\}_{i\in\{1,n\}}$ ordered by increasing magnitude, and V the associated right eigenvectors. We consider the k smallest eigenvalues and V_k the associated right eigenvectors.

Proposition 1. *Let W be such that $\tilde{A}_c = W^H AV_k$ has full rank, $\tilde{M}_c = V_k\tilde{A}_c^{-1}W^H$ and $M_2 = M_1 + \tilde{M}_c$. Then M_2A is similar to a matrix whose eigenvalues are*

$$\begin{cases} \eta_i = \lambda_i & \text{if} \quad i > k, \\ \eta_i = 1 + \lambda_i & \text{if} \quad i \le k. \end{cases}$$

The matrix $\tilde{M}_c$ is defined as the Spectral Low Rank Update (SLRU) for the left preconditioner M_1.

To illustrate the efficiency of this approach we consider a set of large and challenging real life industrial problems. We perform experiments with a parallel fast multipole code [2] to compute the matrix-vector products involving A. For M_1 we choose the preconditioner developed in [3], suitable for implementation in a multipole framework on parallel distributed platforms. It is based on a sparse approximate inverse using a Frobenius norm minimization with an a priori sparsity pattern selection strategy. The spectral information is computed in a preprocessing phase by an external eigensolver: ARPACK [4].

In this talk, we present the gain in terms of times and matrix-vector products, for the complete monostatic calculations [6]. We also illustrate the effects on the convergence rate of GMRES [5] of parameters such as the dimension of the update, the accuracy of the spectral information, the quality of the original preconditioner or the size of the restart. We conclude with some comments on our on-going work where we combine the SLRU preconditioner and the Seed-GMRES or the GMRES-DR [7] solver.

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