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**Error Minimization Based Rezone Strategy for ALE  
Methods**

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The philosophy of the Arbitrary Lagrangian-Eulerian (ALE) methodology for solving multidimensional fluid flow problems is to move the computational mesh, using the flow as a guide, to improve the robustness, accuracy and efficiency of the simulation. The main elements in the ALE simulation are an explicit Lagrangian phase, a rezone phase in which a new mesh is defined, and a remap phase in which a Lagrangian solution is transferred to the new mesh. In the talk we shall address different computational aspects of the rezone phase.

In most ALE codes, the main goal of the rezone phase is to maintain geometrical quality of the mesh. The example of such a strategy is the Reference Jacobian Matrix (RJM) method. Recently we have developed a new Error-Minimization-Based (EMB) rezone strategy which minimizes a global error and maintains smoothness of the mesh. The global error on new time level  $t = t^{n+1}$  is a superposition of an interpolation error, an error due to the time advancing method, and the space discretization error. It is clear that if the mesh is not rezoned, then the interpolation error is zero; however, the mesh at time level  $t = t^n$  may not be the best mesh to represent solution features at time  $t = t^{n+1}$ . The goal of the EMB rezone strategy is to achieve balance (if possible) between the errors which will minimize the global error at time  $t = t^{n+1}$ . The numerical experiments demonstrate the superiority of the EMB rezone strategy over the RJM rezone strategy.

The rezoned mesh is sought as a minimizer of a non-convex functional. The essential part of the talk will be devoted to comparison of efficiency of different non-linear optimization strategies for hyperbolic problems. In particular, we shall consider a few global methods (e.g., a Polak-Ribiere nonlinear conjugate gradient method with inexact line search, a truncated Newton method) and a few local Gauss-Seidel type methods. We shall present numerical results for 1D Burgers' equation and 1D system of gas dynamics equations. The numerical experiments show that the rezone methods perform well even if the rezoned mesh is a local minimum of the functional. It allow us to use inexpensive optimization methods.

Generalization of the EMB rezone strategy for two and three dimensions is a challenging task. It will be addressed quickly in the talk.