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Multigrid Method for Spectral Propagation of Uncertainty in Diffusion Equations

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Stochastic modeling of two-dimensional steady and unsteady diffusion equations in presence of uncertainties in the diffusivity field, forcing term and/or boundary conditions is considered, using spectral expansions. Diffusivity and source fields, as well as the boundary conditions, are stochastic processes which are first represented using their respective Karhunen-Loève (KL) expansions. When truncated, the KL expansions involve a finite set of independent random variables that multiply the eigen-modes of the correlation kernels of the corresponding processes. Propagation and quantification of the impact of these uncertainties on the solution is performed by means of a Polynomial Chaos (PC) expansion, whose spectral coefficients are determined using a Galerkin approach. The Galerkin projection leads to a set of coupled problems for the PC coefficients of the solution which are functions of space and time. Spatial finite-difference discretization of these coupled problems results in a large system of equations, whose dimension necessitates the use of iterative approaches in order to obtain the solution within a reasonable computational time. To accelerate the convergence of the iterative technique, a multigrid method, based on spatial coarsening, is implemented. Numerical experiments show good scaling properties of the method, both with respect to the number of spatial grid points and the stochastic resolution level.