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**Fast Forward Solvers in 3D Diffuse Optical Tomographic  
Imaging**

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Diffuse optical tomographic (DOT) medical imaging makes use of modulated, near-infrared light transmitted into tissue from photodiodes placed on the body. Optical detectors then measure the photon fluence resulting from the scattering and absorption of photons within the region of interest. The goal is to reconstruct three-dimensional images of the scattering and absorption within the tissue. In the case of breast tissue imaging, for example, anomalous regions of absorption and scattering may indicate the presence of a tumor.

We utilize a frequency-domain diffusion equation model for data generation. We parameterize the diffusion (related to the scattering) and absorption in terms of a small number of unknown parameters, represented by the entries in the vector  $p$ . Under the diffusion model, the synthetic data,  $h(p)$ , is a non-linear function of absorption and scattering. If  $y$  represents our measured data then the problem we wish to solve is the weighted, non-linear least squares problem

$$\min_p \|R^{-\frac{1}{2}}(y - h(p))\|_2,$$

where  $R$  is related to the noise in the measured data.

We use a damped Gauss-Newton method to solve the optimization problem. The major difficulty is that computing  $h(p)$  and the entries in the Jacobian requires the solution of several large-scale linear systems. The size of each system depends on the number of voxels in the 3D image. If  $N_s$  denotes the number of sources,  $N_d$  denotes the number of detectors and  $N_w$  is the number of frequencies, the total number of linear systems that must be solved is  $O((N_s + N_d)N_w)$  per Gauss-Newton step. Thus the linear solves are a huge computational bottleneck for the imaging problem.

In this talk, we analyze characteristics of our matrices and discuss techniques for reducing the computational complexity of the forward solves. We consider preconditioned and unpreconditioned Krylov-subspace methods for solving the systems at every outer (i.e. Gauss-Newton) iteration. In particular, we focus

on exploiting relationships among the systems that help minimize the computational effort at a fixed iteration as well as over successive outer iterations.