Jim E. Jones

A Multigrid Solver for Maxwell's Equations Based on Zero Circulation Functions

Center for Applied Scientific Computing
Lawrence Livermore National Laboratory
Box 808
L-561
Livermore
CA 94551
jjones@llnl.gov
David Alber
Barry Lee

In this talk, we consider the curl-curl formulation of the Maxwell's equations, $\nabla \times \mu \nabla \times E + \sigma E = f$, discretized using Nedelec edge elements. Previous researchers' work (Reitzinger and Schoeberl [2002]; Bochev, Garasi, Hu, Robinson and Tuminaro [2003]) on developing fast algebraic multigrid solvers for this equation have used aggregation techniques and carefully designed interpolation to preserve the null-space of the curl operator.

In this talk we describe a new multigrid solver based on element agglomeration (Jones and Vassilevski [2001]). A key component of the method is the notion of a cycle: a closed path of edges in the grid. Coarse grid cycles can be constructed which are a union of fine grid cycles. The multigrid solver then uses an interpolation operator with the property that functions with zero circulation on a coarse cycle are interpolated to have zero circulation on the corresponding fine cycles. This property guarantees that the null-space of the curl operator is preserved (for simply connected domains).

The property of preserving zero circulation functions does not uniquely determine the interpolation operator. Further, this property alone is not enough to yield efficient multigrid solvers. We discuss various methods for defining a unique interpolation operator and present numerical results exploring their effectiveness.