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Preconditioners for volume preserving image registration

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We consider the registration of two images, $R(x)$ and $T(x)$, that is, our goal is to find a reasonable transformation u such that

$$D(u;T,R) = \|T(x+u) - R(x)\|^2 \tag{1}$$

is small.

The problem is ill-posed and therefore, in order to obtain a reasonable transformation we need to integrate prior information about the problem. Some of this information is in the form of the smoothness of the field; such information can be integrated into the regularization of the problem. Thus it is common to find the transformation u by solving the following optimization problem

$$\|T(u) - R\|^2 + \alpha S(u) \ = \ \min \tag{2}$$

where $S(u)$ is usually a quadratic regularization operator and α is a regularization parameter. For example in elastic registration

$$S(u) = \int (\lambda(\mathbf{curl}u)^2 + \mu(\mathbf{div}u)^2) \ dV$$

but many other differential operators can be chosen.

In this work we also assume that the transformation is Volume Preserving (VP). This makes sense in cases when objects may deform but we want to have the same overall volume.

From a mathematical point of view this implies that the transformation also satisfies

$$C(u) = \det(1+u) - 1 \ = \ \mathrm{div}u + N(u) \ = \ 0 \tag{3}$$

where $N(u)$ is some nonlinear operator with terms which depend on the derivatives of u. Thus the optimization problem (2) becomes an equality constraint optimization problem of the form

$$\min \qquad \|T(u) - R\|^2 + \alpha S(u) \tag{4a}$$

$$\text{s.t} \qquad \mathrm{div}u + N(u) = 0 \tag{4b}$$

In this talk we will discuss the KKT systems which evolve from this problem and their numerical solutions. We will show how to use effective multigrid preconditioners for the solution of these systems.