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Quantitative Analysis Methods for Multigrid Solvers

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Limitations of the classical local-mode Fourier (LMF) analysis in application to variable-coefficient problems have been discussed. Some restrictions are required in both the physical and frequency spaces to ensure accurate predictions of convergence in these applications. With these restrictions, certain solution regions, are inaccessible to the LMF analysis.

Alternative, very general, quantitative analysis methods for multigrid solutions of partial differential equation are introduced. The methods are applied to available, non-perfect multigrid solvers that deal with practical problems; parts of the solvers responsible for the less-than-optimal performance are isolated, identified and improved. The analysis methods considered in this talk focus on the main complimentary parts of a multigrid cycle: relaxation and coarse-grid correction. Ideal relaxation (IR) and ideal coarse-grid (ICG) iterations are introduced. In these iterations, one part of the cycle (coarse-grid correction for IR iterations and relaxation for ICG iterations) is actual and its complimentary part is replaced with an ideal imitation. The IR and ICG iterations are very general and can be directly applied in the most complicated situations including highly variable (or nonlinear) coefficients, complex geometries, and unstructured grids. The results of this analysis are not single-number estimates; they are rather convergence patterns of the iterations that may either confirm or refute expectations indicating what part of the actual solver should be improved. The generality of the analysis makes it very valuable tool for analyzing complicated large-scale computational problems, where no other analysis methods are currently available.

Applications of these new analysis methods are demonstrated for model problems arising in computational fluid dynamics.