Edmond Chow

# An aggregation multilevel method based on smoothed error vectors

Lawrence Livermore National Laboratory
L-560
Box 808
Livermore
CA 94551
echow@llnl.gov

We will present a new method for constructing the prolongation operator in aggregation multilevel methods. Suppose a fine grid has been partitioned by nodes into aggregates. For scalar elliptic problems, the error is often represented (roughly) by a piecewise constant function, i.e., the error is represented by a constant function over each aggregate. This constant function is called a basis function for the aggregate.

The performance of methods like the above can be improved for various types of problems by increasing the number of basis functions. This is particularly important when the dimension of the near null-space of the operator is greater than unity, for example in elasticity problems. For elasticity, the rigid body modes are used as basis functions. For other problems, it is not clear how to choose the basis functions, however, they are generally chosen to be smooth functions, e.g., functions of the coordinates of the grid nodes. Related current research has proposed using low-energy eigenvectors of the local stiffness matrices associated with each aggregate.

The prolongation operator $P$ should be able to represent, as well as possible, slow-to-converge error. We thus propose the following method. First, generate $m$ samples of algebraically smooth error by applying the smoother to $Ax = 0$ with a random initial guess. For a given aggregate, let $V$ denote the matrix of $k$ basis vectors being sought, and let $S$ denote the matrix of sample vectors over that aggregate. We seek

$$\min_{V,X} \|VX - S\|.$$

The minimum is achieved when $VX$ is the rank-$k$ matrix nearest to $S$. The matrix $V$ is thus the first $k$ left singular vectors from the singular value decomposition of $S$. The singular values can be used to select $k$. $V$ is used to construct $P$. Note that a different $k$ may be used for each aggregate.

This technique produces basis vectors that are matrix-dependent. In particular, anisotropies and physical jumps in the smoothed error are reflected in the basis vectors. The method can easily be extended to multiple levels. Our experiments show that for an arbitrarily scaled linear elasticity problem, the method can perform as well as if the scaled rigid body modes were known. The method can also be used adaptively, by using V-cycles to generate the smooth error vectors $S$.

This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.