Rafael Bru

Convergence of Additive Schwarz Iterations for Singular Systems

Instituto de Matematica Multidisciplinar
Dept. Matematica Aplicada
Univ. Politecnica de Valencia
46022 Valencia
Spain
rbru@mat.upv.es
Francisco Pedroche
Daniel B Szyld

A convergence analysis is presented for additive Schwarz iterations when applied to consistent singular systems of equations $Ax = b$. The theory applies to singular M-matrices with one-dimensional null space, and is applicable in particular to systems representing ergodic Markov chains. The results are based on an algebraic formulation of Schwarz methods. Let R_i be the restriction operator so that $A_i = R_i A R_i^T$ is a symmetric permutation of a principal submatrix of A (and thus nonsingular). Given an initial vector x^0, additive Schwarz iterations consist of the process $x^{k+1} = T_\theta x^k + c$, $k = 0, 1, \dots$, where

$$T_\theta = I - \theta \sum_{i=1}^{p} R_i^T A_i^{-1} R_i A,$$

$c = \theta \sum_{i=1}^{p} R_i^T A_i^{-1} R_i b$, and $0 < \theta < 1$ is a damping parameter. The key here is of course that one assumes that there is overlap, i.e., that the restriction operators restrict to subspaces with nontrivial intersection. If there is no overlap, the method reduces to block Jacobi.

As A is singular, one has $\rho(T_\theta) = 1$. It is shown that if $\theta < 1/p$ (or more generally $\theta < 1/q$, with q the measure of the overlap), then $\gamma(T_\theta) = max\{|\lambda|, \lambda \in \sigma(T), \lambda \neq 1\} < 1$, and that the index $ind(T) = 1$. Therefore additive Schwarz iterations converge for any initial vector. Furthermore, there exists a splitting $A = M - N$ such that $M^{-1}N = T_\theta$. Our results imply in particular that zero is an isolated point of the spectrum of the preconditioned matrix $\sigma(M^{-1}A)$ and the rest is inscribed in a circle centered at one with radius $\gamma = \gamma(T_\theta) < 1$.

This work complements the results of [Marek and Szyld, *LAA*, in press], where multiplicative Schwarz iterations are shown to converge for singular systems.

(joint work with Francisco Pedroche, Univ. Politecnica de Valencia, Spain, and Daniel B Szyld, Temple University, Philadelphia)