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Multigrid, Mixed Finite Element Methods, And Saddle-Point Problems

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Lowest-order Raviart-Thomas mixed finite element approximations to second-order elliptic PDEs result in a linear system which is symmetric and indefinite (saddle-point problem). The mixed system of equations can be transformed into coupled symmetric positive definite matrix equations, or a Schur complement problem, using block Gauss elimination. Nested iteration and preconditioned conjugate gradient algorithms are the simplest methods for solving the Schur complement problem. The mixed finite element method is closely related to the cell-centered finite difference scheme for solving second-order elliptic problems with variable coefficients. Simple but effective cell-centered multigrid methods have been developed by others in the past. We exploit these methods for solving subsurface flow problems on three-dimensional (possibly distorted) hexahedral elements with discontinuous hydraulic-conductivity coefficients (possibly anisotropic). There are some questions that remain open concerning how to solve problems with large amounts of anisotropy on distorted grids.