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The effect of non-optimal bases on the convergence of Krylov subspace methods

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There are many examples where non-orthogonality of a basis for Krylov subspace methods arises naturally from the application. Similarly, in many occasions it is desirable not to orthogonalize with respect to all previous vectors, thus obtaining truncated bases. One such example is when A is symmetric positive definite, and we use a preconditioner $P = LU$. The preconditioned matrix $L^{-1}AU^{-1}$ is nonsymmetric. One option is to use an "optimal" method for the preconditioned problem, say GMRES or FOM. Another option is to use symmetric Lanczos, i.e., orthogonalizing only with respect to the last two vectors of the basis, thus obtaining a "non-optimal" basis. This is less expensive, but the convergence is "delayed." We explore the question on what is the effect of having this non-optimal basis. In particular, we compare the residuals in the two cases. We conclude that the orthogonality of the basis is not important. We provide a bound on the "delay" which depends on the linear independence of the basis vectors. Numerical examples illustrate our theoretical results.