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Stochastic iterative methods for PDEs with random parameters

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We consider the numerical solution of elliptic or parabolic partial differential equations with stochastic coefficients. Such equations appear in reliability problems. Various approaches exist for dealing with the uncertainty propagation question: Monte Carlo methods, perturbation techniques, variance propagation, etc. Here, we deal with the stochastic finite element method (SFEM) [1]. This method transforms a system of PDEs with stochastic parameters into a stochastic linear system $Ku = f$, by means of a finite element Galerkin discretization. In the classical SFEM-approach, the finite element solution vector u is approximated by a linear combination of deterministic vectors; the coefficients are orthogonal polynomials in the random variables that occur in K and f. Unlike commonly used methods such as the perturbation method, SFEM gives a result that contains all stochastic characteristics of the solution. It also improves Monte Carlo methods significantly because sampling can be done after solving the system of PDEs.

Here, we will show that applying a linear iterative solver (e.g. Gauss-Seidel) to the discretized stochastic system also allows rational instead of polynomial approximations. The computational efficiency can then be improved by using the chosen iterative method as a smoother within a multigrid context, adapting classical ingredients of multigrid methods to the case of rational expressions in random variables. The coefficients in the resulting expressions can differ slightly, however, depending on the amount of information, concerning the random variables, that is taken into account. We will also show how the method can be used efficiently in the case of large stencil discretizations.

References:

[1] R.G. Ghanem and P.D. Spanos. Stochastic finite elements: a spectral approach. Springer-Verlag, New York, 1991.