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Some Characteristics of Multigrid Performance for the Two-Dimensional Convection-Diffusion Equation

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The development of efficient numerical solution techniques for convection-diffusion problems is an important area of current research in the field of iterative methods. As well as being of interest in their own right, convection-diffusion problems are closely linked to the Navier-Stokes equations governing incompressible fluid flow which are widely applicable in industrial settings. One possible approach which has been successfully applied in practice is to use a multigrid method. However, unlike with linear self-adjoint elliptic boundary value problems, the development of related convergence analysis for the convection-diffusion problem has to date been limited. In addition, much of the published theory in the area is very technical and can be hard for the non-expert to interpret. The aim of this talk is to develop a 'simple to use' analysis of multigrid convergence factors for the two-dimensional convection-diffusion equation.

We will focus on the two-grid solution of the system of linear equations arising from a bilinear finite element discretisation, with streamline diffusion added when stabilisation is necessary. Most usually, multigrid convergence is examined in terms of the behaviour of the norms of matrices obtained by splitting the iteration matrix into two parts, representing the approximation property and smoothing property respectively. In this work, we follow this approach while incorporating a matrix transformation which reduces the underlying discretisation matrices to tridiagonal form. This simplifies the analysis, and also enables cheaper numerical computation of norm bounds for large problems. We will demonstrate this technique using a periodic variant of a model problem and use the results obtained to illustrate which trends do and do not agree with those seen when solving (more practically relevant) Dirichlet problems.