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Bifurcation Methods for Large-Scale Problems

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Computing families of solutions to nonlinear problems and their bifurcations can provide invaluable insight to understanding the dynamics of physical and biological systems. However, developing continuation and bifurcation tracking algorithms that are scalable to a large-scale context is quite challenging. In particular, limited memory, lack of derivative information, inexact linear solves from iterative linear solvers, and an inability to add rows and columns to the system Jacobian matrix severly restrict the algorithmic choices available.

In this talk, we will discuss how these difficulties are overcome in LOCA: The Library of Continuation Algorithms, a software package for large-scale continuation and bifurcation tracking developed at Sandia National Laboratories, focusing on the turning point or saddle-node bifurcation commonly observed in nonlinear systems. This algorithm has the advantage of requring little additional information from the application code than it already most likely provides, and only requires linear solves of the underlying system Jacobian matrix. However, these linear solves become extremely ill-conditioned near the turning point bifurcation, limiting the accuracy and robustness of the turning point tracking algorithm. We will then describe a promising new algorithm recently developed at Sandia based upon the idea of applying the Moore-Penrose pseudo-inverse near the bifurcation point. This algorithm avoids ill-conditioned matrix solves and has shown to be significantly more robust and accurate than the original turning point tracking algorithm. Its implementation requires a linear solver with a "matrix free" mode that only requires matrix-vector and preconditioner-vector products. Results from applying these techniques to physical systems of practial interest will be presented.