Dimitri J. Mavriplis

Solution of High-Order Discontinuous Galerkin Methods using a Spectral Multigrid Approach

Department of Mechanical Engineering
Dept. 3295
1000 E. University Ave.
University of Wyoming
Laramie
WY 82071
mavripl@uwyo.edu

The use of high-order Discontinuous Galerkin methods has become more widespread in recent years. These methods are attractive due to their compact support and ease of implementation, particularly for high-order approximations (higher than second order). While much work has been performed in developing spatial discretizations based on this approach, relatively few investigations have addressed the issue of efficient solvers for the resulting discretized equations, with most applications relying on explicit time-stepping approaches.

In the interest of achieving fast steady-state and time-implicit solvers for high-order Discontinuous Galerkin (DG) discretizations, we propose the use of a multigrid method where the coarser levels are obtained by reducing the order of the DG approximation (reducing the spectral index "p") as opposed to the traditional multigrid approach of reducing the grid spatial resolution (discretization index "h"). An "element Jacobi" iterative method is used to drive the "p"-multigrid algorithm. This technique consists of (directly) inverting the block Jacobian associated with all degrees of freedom within each grid cell at each iteration.

Numerical experiments on the linear two-dimensional wave equation demonstrate order-independent convergence rates (for p=1,2,3,4) using element Jacobi as a solver, while convergence degradation with increasing spatial grid resolution (h-refinement) is observed. When the element Jacobi solver is used as a smoother on each level of a "p"-multigrid approach, order independent convergence rates are retained, and rates which are nearly independent of the spatial grid resolution are observed. Future work is underway to improve the grid independence of this approach, and to extend the method to more coplex systems of equations, such as the Euler and Navier-Stokes equations.