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LL* method for Eddy current problem in 3D with edges.

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article

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Maxwell's equations are a set of fundamental equations governing all macroscopic electromagnetic phenomena. The equations can be written in both differential and integral form, but we present them only in simplified differential form, so called Eddy current problem. Here we consider the following two basic laws of electricity and magnetism:

$$\begin{array}{ll} \text{Faraday's Law} & \dfrac{\partial \mu \boldsymbol{H}}{\partial t} + \nabla \times \boldsymbol{E} = 0 \\ \text{Ampère's Law} & \nabla \times \boldsymbol{H} - \sigma \boldsymbol{E} = 0. \end{array}$$

Here we consider the LL^* method to solve least squares of the first order system for partial differential equations. The main goal of the LL^* approach was to use a first order system least squares method for partial differential equations, which do not fulfill the regularity requirements of the standard first order system least squares method. The main idea of the first order system least squares method with the first order equations $L\boldsymbol{U} = \boldsymbol{F}$ is minimizing the functional $||L\boldsymbol{U} - \boldsymbol{F}||_0$, whose bilinear part is equivalent to the product H^1 norm. The standard FOSLS method approximates to the unknown $\boldsymbol{U}$ in the given H^1 finite element spaces. But this H^1-equivalence is provided only under sufficient smoothness assumptions on the original problem like the domain, coefficients, and data. LL^* method was introduced to overcome the problem of standard FOSLS coming from lack of sufficient smoothness by solving the dual problem $L^*\boldsymbol{W} = \boldsymbol{U}$ with the dual variable $\boldsymbol{W}$ and the adjoint operator L^*. So the original problem is recast as one of minimizing the functional $||L^*\boldsymbol{W} - \boldsymbol{U}||_0$ which has the same minimizer of the functional $||L^*\boldsymbol{W}||^2 - 2\langle \boldsymbol{W}, \boldsymbol{F}\rangle$.
FOSLL* has been successfully performed to achieve an accurate approximation using H^1-conforming finite element spaces for the PDE having discontinuous coefficients or irregular boundary points in the bounded plane. But that known method is hard to apply for 3D singular problems specially for the case having

irregular boundary conditions. So we focus on the problem in the domain having irregular boundaries, i.e., reentrant corners. If domain is bounded and either boundary is $\mathcal{C}^{1,1}$ or domain is a convex polyhedron that is given some boundary conditions, then $H(\nabla\cdot) \cap H(\nabla\times)$ is continuously imbedded into $H^1(\Omega)^3$. Since boundary has reentrant corners, the original solution $\boldsymbol{U}$ is not anymore in H^1 and also the dual solution $\boldsymbol{W}$ is not in H^1 which cannot be approximated by H^1 finite element spaces. That is, in the case that the domain has edges, the standard finite element method loses its global accuracy because of the singularities on the boundary. We modify the dual operator L^* and look for a sequence in H^1 converging to the dual solution $\boldsymbol{W}$. Here we don't care the injectivity of L^* because we want to find a solution $\boldsymbol{W}$ for $L^*\boldsymbol{W} = \boldsymbol{U}$.