Henrik Brandén

A Fundamental Solution Preconditioner for First Order PDE

Uppsala University
Department of Information Technology
Box 337
SE-751 05 Uppsala
SWEDEN
Henrik.Branden@it.uu.se
Per Sundqvist
Sverker Holmgren

We present a new preconditioner for the iterative solution of systems of equations arising from discretisations of systems of first order partial differential equations on structured grids. Such systems occur in many important applications, including compressible fluid flow and electromagnetic wave propagation.

The preconditioner is a truncated convolution operator, with a kernel that is a fundamental solution of a difference operator closely related to the original discretisation. We give an FFT-based algorithm for computing fundamental solutions.

Analysis of a relevant scalar model problem in two spatial dimensions shows that grid independent convergence is obtained using a simple one-stage iterative method.

As an example of a more involved problem, we consider the steady state solution of the non-linear Euler equations in a two dimensional, narrowing channel. We present results from numerical experiments, verifying that the preconditioning technique again achieves grid independent convergence, both for an upwind discretisation and for a centred second order discretisation with fourth order artificial viscosity.