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**Preconditioning Techniques in TRANAIR: an Industrial
Application**

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TRANAIR is a general geometry computational fluid dynamics (CFD) tools used by Boeing Commercial Airplanes for aerodynamic design, optimization, and analysis and is run thousands of times a year. It is applied to large scale problems containing millions of variables and is Boeing Commercial main CFD “work horse”. It can be used to analyze or design aerodynamic configurations using the nonlinear full potential equation with a coupled integral boundary layer. In the far field the full potential equation reduces to the linear Prandtl-Glauert equation. Unsteady TRANAIR computes the effects of a linear harmonic perturbation about the steady velocity field, representing a flow solution about an aircraft configuration. In the far field the latter reduces to the convective Helmholtz equation.

For exterior flow problems the far field boundary condition is a non-trivial issue that needs careful care. To achieve high accuracy a charge formulation is implemented in TRANAIR. Assuming the flow is linear at the far field, the charge formulation amounts to having the far field boundary at infinity. The variables are split into two major sets: global grid (X) and refined grid (Y) variables. The global grid variables are transformed to charge variables by $X = T^{-1}Q$, where T is a linear operator that is fast to invert, and Q is different than zero only where the non linear operator, L , is not equal to T . This approach is reminiscent of a two level multigrid method where Q and T are the “coarse grid” variable and operator respectively. An inexact Newton solver in combination with GMRES (Generalized Minimum Residual Method) is applied to this problem taking advantage of the fact that it is very inexpensive to multiply a vector of Q by T^{-1} and by L . The calculation of the preconditioned residual, R , is computed by $R = N^{-1} \times L \times T^{-1} \times Q, Y]$. The left preconditioning matrix, N , is approximating L excluding the far field. It is large and sparse and therefore it is feasible to apply a direct sparse incomplete factorization of N . A drop tolerance

can be introduced into the sparse elimination process allowing small elements in the decomposition to be dropped as they are generated. A grid based nested dissection ordering is generated, which reduces fill during elimination.

Most preconditioning techniques assume a given matrix that represents the linear system at hand. One difficulty with the far field charge formulation approach is that the full Jacobian matrix of the linearized system is not accessible, limiting the scope of preconditioning opportunities (such as right preconditioning). Recently, a new approach has been developed in which the charges are eliminated and instead a discrete integral equation is applied at the far field. That equation was obtained by calculating the discrete Green's identity in the domain between the computational box and infinity using the discrete Greens function. The resulting Jacobian matrix is mostly sparse but significantly denser at the far field. Given a uniform N^3 grid in 3 space dimensions, $O(N^2)$ of the Jacobian rows will have $O(N^2)$ elements while the remaining rows will have $O(1)$ elements.

In the talk recent preconditioning results using TRANAIR will be presented, focusing on the role of the far field condition in exterior flow. Among the issues that are addressed are right and left preconditioning, and reordering schemes to account for the denser parts of the Jacobian matrix.