Patrick Greene

Newton's Method with Discontinuous Galerkin Projections Applied to Minimization Problems in Level Set Re-distancing and Weighted Condition Number Mesh Relaxation

P O Box 808
L-405
Livermore
CA 94551
greene30@llnl.gov
Robert Nourgaliev
Sam Schofield

We demonstrate how Newton's method can be effectively combined with discontinuous Galerkin projections for minimization problems based on discretized quantities. Utilizing a discontinuous Galerkin projection provides a simple method to compute the derivatives needed for the Jacobian and Hessian since the derivatives are simply evaluated from the prescribed basis functions. The resulting projected function and its derivatives can then be used with Newton's method to solve the minimization problem. Two applications of the proposed method are shown.

The first application of the method is for re-distancing in a high-order unstructured mesh level set method. The re-distancing process requires finding the shortest distance from a given point to the zero contour of the level set. We introduced a new re-distancing algorithm, where this process is formulated as a constrained minimization problem with the level set represented as a discontinuous Galerkin projection. Static and dynamic front evolution results will be shown, confirming that the computed distance function maintains the same order of convergence as the level set projection.

The second application is weighted condition number (CN) mesh relaxation. CN mesh relaxation is used in arbitrary Lagrangian-Eulerian (ALE) simulations to prevent mesh tangling, which can result from the Lagrangian mesh motion. The method works by defining a condition number functional at each vertex, which is based on the near by cell geometries, and then iteratively minimizing the functional to find new vertex positions. The inclusion of a weight allows the mesh relaxer to concentrate mesh density near regions of interest. A new formulation of the weights using a discontinuous Galerkin projection is presented. The new method can cluster grid cells near interfaces prescribed by discrete cell-centered values, such as an index function or volume fraction. Sample meshes demonstrating the methods ability to cluster cells around dynamically evolving

complex geometries are given.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. Information management release number LLNL-ABS-681162.