Luis Garcia Ramos

On the spectrum of deflated matrices with applications to the deflated shifted Laplace preconditioner for the Helmholtz equation

Technische Universität Berlin
Institut für Mathematik
Straße des 17 Juni 136
10623 Berlin
Germany
garcia@math.tu-berlin.de
Reinhard Nabben

The deflation technique to accelerate Krylov subspace iterative methods for the solution of linear systems has been known for a long time. The first landmark papers are due to Nicolaides and Dostál in the late eighties where deflation is used for Hermitian positive definite (HPD) linear systems. In the last decade deflation was used and analyzed in combination with domain decomposition and multigrid methods, resulting in very effective algorithms. Examples are the multilevel Krylov methods introduced in [1,2], see also [3,4] where multilevel deflation techniques are presented. Although these algorithms work very well in practice for non-Hermitian problems, not many theoretical results are known so far in this direction. Here we show inclusion regions for the spectrum of an arbitrary deflated matrix based on the field of values, which generalize known results for HPD systems. Moreover, for deflated GMRES we show a residual bound based on the field of values. We apply our results to linear systems arising from the Helmholtz equation. We focus on the combination of the complex shifted Laplace (CSL) preconditioner [5] with the multilevel Krylov technique. Numerical examples indicate that the eigenvalues of the deflated CSL-preconditioned system lie on exact the same circles as the CSL-preconditioned linear systems and are shifted away from zero. Here we are able to prove these surprising results for any wavenumber and any dimension using Möbius transformations and the Spectral Mapping Theorem. Our new results help to explain the good performance of multilevel Krylov methods for the Helmholtz equation.

References

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