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Kevin Carlberg  
**Applying model reduction to Krylov-subspace recycling:  
the POD-augmented conjugate-gradient method**

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This talk presents a new Krylov-subspace-recycling method for efficiently solving sequences of linear systems of equations characterized by varying right-hand sides and symmetric-positive-definite matrices. As opposed to typical truncation strategies used in recycling such as deflation, we propose a truncation method based on a technique from nonlinear model reduction: goal-oriented proper orthogonal decomposition (POD).

This idea is inspired by the observation that model reduction aims to compute a low-dimensional subspace that contains an *accurate* solution; as such, we expect the proposed method to generate a low-dimensional subspace that is well suited for computing solutions that can satisfy *inexact* tolerances. In particular, we propose specific goal-oriented POD ‘ingredients’ that align the optimality properties of POD with the objective of Krylov-subspace recycling.

To efficiently compute solutions in the resulting ‘POD-augmented’ Krylov subspace, we propose a novel hybrid direct/iterative three-stage method that leverages 1) the optimal ordering of POD basis vectors, and 2) well-conditioned reduced matrices. Numerical experiments performed on solid-mechanics problems highlight the benefits of the proposed method over existing approaches for Krylov-subspace recycling.