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Algebraic Multigrid Preconditioners for Multiphase Flow in Porous Media

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Multiphase flow is a critical process in a wide range of applications, including carbon sequestration, contaminant remediation, and groundwater management. It is modeled by a nonlinear system of partial differential equations derived by considering the mass conservation of each phase (e.g., oil, water), along with constituitive laws for the relationship of phase velocity to phase pressure. The constraint that the phase saturations sum to one, along with initial and boundary conditions, closes this system. The nonlinearity of the constuitive laws, in conjuction with the coupling of the phases, often requires the use of implicit discretization in time for both stability and accuracy. In this work we study a model of immiscible two-phase flow in which the primary variables are the pressure of the wetting phase and the saturation of the nonwetting phase. We use a finite volume method for spatial discretization and the backward Euler method for time discretization of the coupled system, leading to a fully implicit solution method. In this setting the capillary pressure is the difference between the nonwetting phase and wetting phase pressures. If the capillary pressure changes quickly with respect to saturation, then the operators associated with each phase are diffusion-dominated, whereas if capillary pressures vary slowly, then the saturation depends strongly on an advection-dominated, nearly hyperbolic, operator. These variations in character affect the performance of AMG-based solvers. Here, we present our experience with the GMRES solution of the linear systems resulting from the linearization of the coupled equations with algebraic multigrid (AMG) as a preconditioner, and with two constrained pressure residual multigrid (CPR-AMG) preconditioners. AMG is implemented using the HYPRE software package from Lawrence Livermore National Laboratories. Numerical expermients demonstrate that GMRES with AMG preconditioning for the coupled systems works best in the diffusion-dominated case, but it suffers slow convergence in the advection-dominated case. Similarly, the previously established CPR-AMG method that uses AMG to solve the pressure block, do not scale optimally with problem size in the advection-dominated case. The proposed CPR-AMG method that uses AMG to solve both the pressure block and the saturation block performs well for advection-dominated problems, and scales optimally with problem size.