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A Certified Trust Region Reduced Basis Approach to PDE-Constrained Optimization

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Parameter optimization problems constrained by partial differential equations (PDEs) appear in many science and engineering applications. In such problems, the PDE usually describes the underlying system behavior, while the parameters specify particular configurations of the system, such as boundary and initial conditions, material properties, and geometry. If classical discretization techniques (e.g. finite element or finite volume analysis) are used, solving these optimization problems may require a prohibitively large number of computationally expensive PDE solves, especially if the number of parameters to be optimized is high. One way to decrease this computational burden is through the use of surrogate models, where the original high-dimensional model is replaced by an approximation of reduced dimension.

Numerous model reduction approaches exist in the literature and have been successfully applied to PDE-constrained optimization problems. In this work, we build on the reduced basis (RB) method, a model reduction method which, in addition to yielding efficient reduced-order approximations for a large class of parameterized PDEs, also allows efficient computation of a posteriori error bounds on the approximate solution. Traditionally, RB models are generated for a specified parameter space during a computationally-expensive offline phase, using the error bounds to guarantee accuracy of the approximation over this entire space before the reduced model is used online for efficient optimization. However, this approach has the drawback of expending significant computational effort that turns out to be unneeded, because surrogate model accuracy is required only along the optimization trajectory, rather than across the entire admissible parameter space.

This work breaks from the traditional offline-online RB decomposition to progressively build the reduced basis model within a trust region optimization framework. We note that there are several contributions on the use of approximate models in a trust region framework, including works by Alexandrov et al., Arian et al., Zahr and Farhat, and others. In this talk, we present the combined trust region reduced basis framework and guarantee its convergence based solely on the error bounds, using a recent result by Yue and Meerbergen which replaces the standard first-order conditions with relaxed first-order conditions. The proposed certified trust region RB approach requires only a minimal number of high-order solves, used to update the RB model if the approximation is no longer sufficiently accurate. We consider problems governed by elliptic PDEs and present numerical results for a thermal fin model problem with up to six parameters.