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**An optimal, fully implicit algorithm for the low-beta
extended MHD model**

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The low- β extended magnetohydrodynamics (low- β XMHD) model is obtained by taking the large-guide-field and cold-ion limit of the extended MHD model. The resulting model is appealing owing to its simplicity (it is a small set of scalar equations), and because it describes a wide range of laboratory magnetic confinement devices, the solar corona, and other astrophysical plasmas in which large guide fields are present.

However, the numerical integration of the low- β XMHD system is non-trivial due to the presence of disparate time and length scales, which demand both spatial adaptivity and efficient implicit integration methods for efficiency. The large time-scale disparity originates in the presence of fast dispersive waves, which result in significant numerical stiffness and the need of high-order dissipation operators to prevent numerical noise in nonlinear regimes. Both dispersive hyperbolic systems and high-order differential operators stress numerical algorithms, and benefit from an implicit treatment.

Despite the relevance of the low- β XMHD system in the study of magnetized plasmas, to our knowledge there is scant effort devoted towards the development of modern, efficient implicit algorithms for the numerical solution of the low- β XMHD model. There are several efforts in the literature record that employ implicit timestepping,^{1,2,3} but only the latter reference makes some effort to characterize the solver performance. It employs a Newton-Krylov-Schwarz implicit parallel solver, with incomplete ILU methods with various degrees of overlap in each parallel domain. Performance is quite sensitive to the domain overlap, and iteration count is quite high, but scales reasonably well in parallel. Reported speedups with respect to explicit approaches are at most of an order of magnitude for a 1980×1980 mesh.

The focus of this paper is to demonstrate an efficient, optimal nonlinearly im-

¹G. T. A. Huysmans, *Plasma Phys. Control. Fusion*, **47** (2005)

²K. Germaschewski, A. Bhattacharjee, and C.-S. Ng, “The magnetic reconnection code: an AMR-based fully implicit simulation suite,” in *Numerical Modeling of Space Plasma Flows* (N. B. Pogorelov and G. P. Zank, eds.), vol. 359 of ASP Conference Series, 2006.

³S. Ovtchinnikov, F. Dobrian, X.-C. Cai, and D. Keyes, *J. Comput. Phys.*, **225** (2007).

plicit algorithm for the low- β XMHD model. The approach uses Jacobian-free Newton-Krylov (JFNK) methods, effectively preconditioned using physics-based approximations of the Jacobian system that are multigrid-friendly, and therefore deliver optimal convergence rates. The preconditioning approach presented here leverages earlier developments of effective physics-based preconditioners for MHD⁴ and extended MHD,⁵ and in particular employs a similar parabolization strategy to that presented in these studies. We demonstrate the performance of the algorithm with challenging numerical examples. In particular, we demonstrate optimal scaling under mesh refinement, and CPU speedups with respect to explicit methods of 2 to 3 orders of magnitude even for moderate meshes (256×256). We apply the algorithms to the problem of fast reconnection in the large-guide-field regime to derive new physical insight for this challenging problem.

⁴L. Chacn, D. A. Knoll, and J. M. Finn, *J. Comput. Phys.*, **178** (2002)

⁵L. Chacn and D. A. Knoll, *J. Comput. Phys.*, **188** (2003)