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**Block Solvers for Implicit Electromagnetic and Plasma  
Simulations**

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Multiple time-scales can arise in electromagnetic simulations when heterogeneous material properties result in multiple light speeds and when currents are applied at slower time-scales than the speed of light. When time-scales of interest are slower than the fastest time-scales admitted by the model, yet the displacement current cannot be ignored, implicit time integration of the full Maxwell equations can be an effective and competitive means of electromagnetic simulation. In this talk, we consider a compatible discretization of the Maxwell equations, using edge elements for the electric field and face elements for the magnetic induction. This discretization results in discrete enforcement of the solenoidal involution  $\text{div } \mathbf{B} = 0$  to machine precision. In order to take advantage of existing solver technology, we propose block preconditioners in which edge and face degrees of freedom are segregated. These preconditioners use Schur complement approximations to capture the stiff off-diagonal physics. We design these approximations so that either specialized edge element AMG solvers or more traditional multigrid methods can be employed for component solves. We continue by applying these techniques to a fully coupled multi-fluid plasma simulation in which the compatible Maxwell discretization is coupled to a nodal hydrodynamic discretization. We employ block factorizations that allow the combination of our new Maxwell solvers with state of the art Navier-Stokes solvers. We demonstrate robustness and scalability of our methods on a sequence of difficult test problems.