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Smoothed aggregation based AMG preconditioning of distributed optimal control problems constrained by elliptic equations based on smoothed aggregation

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In this work we propose a new algebraic multigrid (AMG) method for the reduced form of linear-quadratic optimal control problems constrained by elliptic equations of the form

$$\min_{y,u} \frac{1}{2}\|y - y_d\|^2 + \frac{\beta}{2}\|u\|^2, \quad \Delta y = u \ . \tag{1}$$

Previous work has shown that problems like (1) can be solved very efficiently using multigrid methods, assuming a geometric hierarchy of grids is available. As with geometric multigrid, AMG methods have been originally designed for solving linear systems representing discrete differential equations, where the matrices are sparse. Instead, the systems arising in the reduced (1), the so-called reduced Hessians, are not sparse, therefore AMG methods are not directly applicable. Relying on the basis functions and prolongation operators originating in smoothed aggregation for the elliptic equation, we construct an AMG-based multigrid preconditioner for (1) that exhibits an efficiency similar to the geometric multigrid case, namely it increases with increasing resolution.