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Ruxin Dai  
**A high accuracy high efficiency solution for anisotropic  
Poisson equation**

Department of Computer Science and Information Systems  
University of Wisconsin River Falls  
410 S Third Street  
River Falls  
WI 54022  
USA  
`ruxin.dai@uwrf.edu`  
Ruxin Dai  
Pengpeng Lin

The solution of anisotropic Poisson equation appears in many computational science and engineering and industry modeling and simulation applications. Therefore, a study of high accuracy and high efficiency solution for anisotropic Poisson equation is carried out. Taking into account the feature of anisotropic, unequal mesh size discretization is used, in which more grid points are distributed in the direction which changes fast while less grid points are distributed in the direction which changes slowly. Although there exists fourth order compact difference scheme on unequal mesh size grids for anisotropic Poisson equation, this study aims to develop a more cost-effective method which can achieve sixth order of accuracy on computed solutions. The gain in accuracy is obtained through applying completed Richardson extrapolation on two computed fourth order solutions from different scale grids with unequal mesh sizes. In this work, theoretical analysis is conducted to show that the Richardson extrapolation is able to remove the heading truncation error terms from unequal mesh size discretization and thus to improve the order of accuracy. In order to improve computational efficiency, partial semi-coarsening multigrid method is adopted to solve the resulting linear systems and multiscale multigrid computation is involved to speed up the whole solution process. Numerical experiments are conducted to test the accuracy and efficiency of the proposed method and to compare it with the existing fourth order method.

**Keywords:** Anisotropic Poisson equation, Richardson extrapolation, unequal mesh size, partial semi-coarsening, multigrid method, multiscale multigrid computation.