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**Physics-based pre-conditioners for large-scale subsurface
flow simulation**

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In this paper we consider new physics-based pre-conditioning techniques for solving large, sparse linear systems of equations with strongly varying coefficients and localised source terms. Such systems occur during the simulation of multi-phase flow through strongly heterogeneous porous media as occur in e.g. oil and gas production, CO₂ storage or (contaminated) ground water flow. The underlying partial differential equations are typically of two different types: a weakly nonlinear near-elliptic parabolic equation with time-varying parameters to describe the diffusive pressure behaviour, and one or more strongly nonlinear nearly-hyperbolic parabolic equations to describe the mostly convective behaviour of the interfaces between the phases. The spatially discretized coupled nonlinear equations are typically solved with a low-order time discretization and Newton-Raphson iteration.

The state of the art approach to solve these systems is to apply a two-stage preconditioning in which first the pressure-related part of the matrix is isolated (with the aid of Schur decomposition) and solved, typically with an Algebraic Multi Grid (AMG) solver. Thereafter the full system of equations is solved with e.g. a Conjugate Gradient solver. We consider the first step of this two-step procedure, i.e. solving of the pressure equation. A particular aspect of porous media flow is the relatively small number of wells which are usually positioned quite far apart. This results in pressure solutions that are dominated by the near-well pressure behaviour in addition to the geological spatial features.

In [1] it is shown that an effective alternative to the AMG solver for the pressure equation can be provided by a reduced-order model obtained by applying proper orthogonal decomposition (POD) to a number of pre-computed solutions. In case of optimisation or parameter estimation exercises which require tens to hundreds of 'reservoir simulations' of nearly identical models with nearly identical right-hand sides (source terms), the CPU time to compute the reduced-order model is negligible.

In this paper we analyse and further develop the preconditioners given in [1]. We investigate alternatives using different varieties of reduced-order modelling. Furthermore we explore the connection between POD-based preconditioning and deflation methods. One of the difficulties for deflation methods is to find the right deflation vectors for general problems. A start has been made to find these vectors in an automated way, however until now this remains an open question. The combination of deflation with the POD methods looks very promising in this respect.

Some numerical experiments are given to illustrate the theory. We start with a simple layered problem and also give results for the SPE10 benchmark [2].

References

- [1] Astrid, P., Papaioannou, G.V., Vink, J.C. and Jansen, J.D., 2011: Pressure preconditioning using proper orthogonal decomposition. Paper SPE 141922 presented at the SPE Reservoir Simulation Symposium, The Woodlands, USA, 21-23 February
- [2] <http://www.spe.org/web/csp/datasets/set02.htm>