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Brian Weston  
**Physics-Based Preconditioning for a Newton-Krylov  
Framework in a High-Order rDG-based Navier-Stokes  
Solver**

Lawrence Livermore National Laboratory (LLNL)  
Livermore  
CA 94551  
`weston8@llnl.gov`  
Robert Nourgaliev  
Jean-Pierre Delplanque

We investigate the preconditioning of an all-speed Navier-Stokes solver, based on the orthogonal-basis reconstructed Discontinuous Galerkin (rDG) method, and integrated using high-order, fully implicit time discretization schemes. The work is motivated by laser-induced phase change applications, particularly the selective laser melting (SLM) process in additive manufacturing. Simulations of the SLM process require precise tracking of multi-material solid-liquid-gas interfaces, due to laser-induced melting/solidification and evaporation/condensation of metal powder in an ambient gas. These rapid density variations and phase change processes tightly couple the governing equations, requiring a fully compressible framework to robustly capture the rapid density variations of the ambient gas and the melting/evaporation of the metal powder.

The governing equations are discretized up to 4th-order accuracy with our rDG spatial discretization scheme and up to 5th-order accuracy with L-stable fully implicit time discretization schemes (BDF2 and ESDIRK3-5). The resulting set of non-linear equations is solved using a robust Newton-Krylov method, with the Jacobian-free version of the GMRES solver for linear iterations. Due to the stiffness of the underlying physics associated with fast acoustic waves and thermal and viscous/material strength effects, GMRES must be preconditioned. To accelerate the convergence of the Newton-GMRES solver, we employ a semi-implicit, physics-based operator split preconditioner (PBP) with a p-multigrid approach. While physics-based operator-split preconditioning has been successfully deployed in the past for finite volume based Navier-Stokes solvers, using a p-multigrid technique for the preconditioning of DG-based solvers is relatively new, especially in the context of the Jacobian-free Newton Krylov (JFNK) methodology.

We investigate different options to split the governing equations for the linearized residuals, rendering a reduced set of the approximate Jacobian as the preconditioning matrix. In addition, we explore different stationary iterative methods as well as the ordering of the physics (block) degrees of freedom. We

demonstrate that our JFNK-PBP framework converges for high CFL numbers on classic benchmark problems in fluid dynamics (lid-driven cavity flow and natural convection heat transfer) as well as for laser-induced phase change problems in 2D and 3D.

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