Andy Wathen A preconditioner with guaranteed rapid convergence for nonsymmetric real Toeplitz systems

Mathematical Institute
Woodstock Road
Oxford
UK
wathen@maths.ox.ac.uk
Jen Pestana

Descriptive convergence estimates or bounds for Krylov subspace iterative methods for nonsymmetric matrix systems are keenly desired but remain elusive. In the case of symmetric (self- adjoint) matrices, bounds based on eigenvalues can be usefully descriptive of observed convergence; an important consequence is that there are rigorous criteria for what constitutes a good preconditioner for symmetric matrices. For nonsymmetric matrices preconditioning must generally be heuristically motivated.

Such comments apply quite generally, however there is one class of nonsymmetric matrices for which we have recently been able to rigorously prove descriptive convergence bounds, namely real Toeplitz (constant diagonal) matrices. Our results apply regardless of non-normality or any 'degree' of nonsymmetry.

Gil Strang proposed the use of circulant matrices (and the FFT) for preconditioning symmetric Toeplitz matrix systems in 1986 and there is now a well-developed theory which guarantees rapid convergence of the conjugate gradient method for such preconditioned positive definite symmetric systems.

In this talk we describe our recent approach which provides a preconditioned MINRES method with the same guarantees for real nonsymmetric Toeplitz systems regardless of the non-normality.