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A Quasi-Algebraic Multigrid Approach to PDE systems that include Material Interfaces and Variable Degrees-of-Freedom Per Node

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The application of algebraic multigrid methods to PDE systems can be problematic, especially when the number of degrees-of-freedom per node varies. This talk considers a multi-phase flow problem corresponding to the movement of an air bubble within a surrounding water body. The incompressible Navier-Stokes equations are discretized via stabilized linear finite elements using a moving mesh that conforms to the bubble/air interfaces. In this formulation, the number of degrees-of-freedom at nodes on the bubble interface is greater than the rest of the domain due to the presence of unknowns for both air and water pressure.

To tackle this problem with multigrid, we consider an approach that applies AMG to a scalar Laplace-like problem on the same mesh as the original problem with the exception that edges between interface nodes and the rest of the domain are removed. The grid transfers for this Laplace problem are then used to build interpolation/restriction operators for the original incompressible flow application. The overall scheme is quasi-algebraic in that coordinates (to build the Laplace-like problem) are required as well as a boolean table indicating which degrees-of-freedom are defined at each nodal location. Numerical results are presented highlighting strengths of this approach. Additionally, alternative algorithms for the construction of grid transfers are also described. These alternatives essentially incorporate more matrix information and are potentially helpful when the underlying PDE coefficients are highly variable within each isolated region.