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Preconditioners for nonsymmetric linear systems with low-rank skew-symmetric part

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In this work we study the iterative solution of nonsingular, nonsymmetric linear systems of n equations

$$Ax = b \tag{1}$$

where the skew-symmetric part of the coefficient matrix A can be approximated by a low-rank matrix. Consider $A = H + K$ where H and K are the symmetric and skew-symmetric parts of A, respectively. It is assumed that the skew-symmetric matrix can be written as $K = PQ^T + E$ for some full rank $P, Q \in \mathbb{R}^{n\times s}$ with $k \ll n$, and $\|E\| \ll 1$.

Different strategies have been proposed when the skew-symmetric part K has exactly rank $s \ll n$. In [1] it is presented a progressive GMRES method that allows for the short-term computation of an orthogonal Krylov subspace basis. As pointed out in [5], although the method is mathematically equivalent to full GMRES [6] in practice it may suffer from instabilities due to the loss of orthogonality between the vectors of the generated Krylov subspace basis. In the same paper, the authors propose a Schur complement method that also permits the application of short-term formulas. The method obtains an approximate solution by applying the MINRES method $s+1$ times. The authors also suggest that can be successfully applied as a preconditioner for GMRES for the problem considered in this paper.

We study a method based on the framework proposed in [4]. Assuming that the matrix $H + PQ^T$ is nonsingular, our approach computes an approximate LU factorization of the matrix

$$\begin{bmatrix} H & P \\ -Q^T & I \end{bmatrix} \tag{2}$$

with the Balanced Incomplete Factorization (BIF) algorithm [2, 3]. Interestingly, the matrix in (2) is similar to the one used in [5] to develop the Schur

complement method, but in this work it is used to update a previously computed preconditioner for the symmetric part H. Then, the factorization is used as a preconditioner for the GMRES method. The results of the numerical experiments for different problems will be presented.

Bibliography

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