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An efficient preconditioned iterative algorithm for Helmhotz wave propagation models

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We consider the Helmholtz acoustic wave propagation model in a bounded media with an inhomogeneous impedance boundary condition on its boundary. It is well known that the standard Galerkin variational formulation of the Helmholtz partial differential equation (PDE) is indefinite for large wavenumbers, and hence the Helmholtz PDE in some literature is known as sign-indefinite. The lack of coercivity (indefiniteness) in the standard Galerkin problem and iterative algorithms associated finite element method (FEM) models is one of the major difficulties for simulating wave propagation models using iterative methods.

A recent theoretical variational formulation of the Helmholtz PDE shows that coercivity property can be obtained using a non-standard approach. However, the authors of the theoretical article also questioned the practical use of their sign-definite formulation even for the constant coefficient Helmholtz equation with impedance boundary condition. In particular, the key open issue is: Whether the recent sign-definite formulation can alleviate some of the associated iterative algorithm.

Through various computer simulations, we provide a concrete answer that the theoretically analyzed sign-definite formulation does not alleviate the key difficulty of reducing the GMRES iterations of the associated FEM model. We subsequently develop a new class of efficient preconditioned iterative Galerkin FEM wave propagation models for the Helmholtz equation.