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Statistically inspired preconditioned iterative solvers for ill-posed linear systems

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The solution of ill-posed linear systems via iterative methods is a versatile tool to account for qualitative features that are believed to characterize the solution when recasting the problem in a probabilistic framework where every unknown is modeled as a random variable. In this talk we show how to use a hierachical model and anatomical features of the brain to design a class of right preconditioners for the solution of the severely underdetermined inverse problem of magnetoenchephalography. These preconditioners depend on a set of parameters that are estimated together with the unknown of primary interest. In the general setup, the parameters can be interpreted as the variances of the entries of the unknown, with a higher parameter value indicating that the corresponding solution entry may have a value significantly different from the nearby entries. The algorithm for estimating the unknown and the parameters in the preconditioner is an iterative alternating scheme (IAS) which updates alternatively one of the two set of unknowns, while keeping the other fixed. The IAS algorithm can be computationally very efficient: the updating of the unknown requires the solution of a least squares problem, while an explicit formula can be used to update the parameters when they are assumed to follow a Gamma distribution. Moreover, the IAS algorithm is globally convergent and it can be shown that one of the two parameters of the Gamma distribution can be tuned to promote sparsity. A statistically motivated criterion for stopping the iteration in the solution of the least squares problems is also presented.