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**Monolithic Multigrid for a B-field Incompressible,
Resistive Magnetohydrodynamics Model**

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Magnetohydrodynamics (MHD) models describe a wide range of plasma physics applications, from thermonuclear fusion in tokamak reactors to astrophysical models. These models are characterized by a nonlinear system of partial differential equations in which the flow of the fluid strongly couples to the evolution of electromagnetic fields. As a result, the discrete linearized systems that arise in the numerical solution of these equations are generally difficult to solve, and require effective preconditioners to be developed. This paper investigates monolithic multigrid preconditioners for a one-fluid, viscoresistive MHD model in two dimensions that utilizes a second Lagrange multiplier added to Faraday's law to enforce the divergence-free constraint on the magnetic field. We consider the extension of a well-known relaxation scheme from the fluid dynamics literature, Vanka relaxation, to this formulation. To isolate the relaxation scheme from the rest of the multigrid method, we utilize structured grids, geometric interpolation operators, Galerkin coarse grid operators, and inf-sup stable elements for both constraints in the system. Numerical results are shown for the Hartmann flow problem, a standard test problem in MHD.