

---

Jeffery Allen  
**Fluidity Based Formulation of Nonlinear Stokes Flow for  
Ice Sheets using FOSLS**

2210 Walnut Apt 1  
Boulder  
CO 80302  
`jeffery.allen@colorado.edu`  
Thomas Manteuffel  
Harihar Rajaram

This talk describes two First-order System Least Squares (FOSLS) formulations of a nonlinear Stokes flow model for glaciers and ice sheets. In Glen's law, the most commonly used constitutive equation for ice rheology, the ice viscosity becomes infinite as the velocity gradients (strain rates) approach zero, which typically occurs near the ice surface where deformation rates are low, or when the basal slip velocities are high. The computational difficulties associated with the infinite viscosity are often overcome by an arbitrary modification of Glen's law that bounds the maximum viscosity. In this talk, two FOSLS formulations (the viscosity formulation and fluidity formulation) are presented. The viscosity formulation is a FOSLS representation of the standard nonlinear Stokes problem. The new fluidity formulation exploits the fact that only the product of the viscosity and strain rate appears in the nonlinear Stokes problem, a quantity that, in fact, approaches zero as the strain rate goes to zero. The fluidity formulation is expressed in terms of a new set of variables and overcomes the problem of infinite viscosity. The new formulation is well posed and the linearization is essentially  $H^1$ -elliptic around solutions for which the viscosity is bounded. A Nested Iteration (NI) Newton-FOSLS-AMG approach is used to solve both nonlinear Stokes problems, in which most of the iterations are performed on the coarsest grid. Both formulations demonstrate optimal finite element convergence. However, the fluidity formulation is more accurate. The fluidity formulation involves linear systems that are more amenable to solution by AMG. Further improvement in computational cost is achieved using local adaptive refinement.