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**Algebraic Multigrid Preconditioners for Multiphase Flow  
in Porous Media**

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Multiphase flow is a critical process in a wide range of applications, including carbon sequestration, contaminant remediation, and groundwater management. It is modeled by a nonlinear system of partial differential equations derived by considering the mass conservation of each phase (e.g., oil, water), in conjunction with constitutive laws for the relationship of phase velocity to phase pressure. The constraint that the phase saturations sum to one, along with initial and boundary conditions, closes this system. The nonlinearity of the constitutive laws in conjunction with the coupling of the phases often requires the use of implicit discretization in time for both stability and accuracy. In this work we present our experience with algebraic multigrid (AMG) and two constrained pressure residual multigrid (CPR-AMG) preconditioners for solving the linear systems resulting from linearization of the coupled equations governing multiphase flow with capillarity effect. We use a finite volume method for spatial discretization and the backward Euler method for time discretization for the coupled system, leading to a fully implicit solution method. AMG is implemented using the HYPRE software package from Lawrence Livermore National Laboratories. Our focus in this study is on two-phase flow in which the primary variables are the pressure of the wetting phase and the saturation of the nonwetting phase. In this model, if capillary pressures change quickly with respect to saturation, then the operators associated with each phase are diffusion-dominated, whereas if capillary pressures vary slowly, then the saturation depends strongly on an advection-dominated, nearly hyperbolic, operator. These variations in character affect the performance of AMG solvers. In particular, AMG for coupled systems works best in the diffusion-dominated case, but it suffers slow convergence in the advection-dominated case.