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On the Preconditioning of a High-Order RDG-based All-Speed Navier-Stokes Solver

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We investigate the preconditioning of an all-speed Navier-Stokes solver, based on the orthogonal-basis Reconstructed Discontinuous Galerkin (RDG) space discretization, and integrated using a high-order fully-implicit time discretization method. The work is motivated by applications in Additive Manufacturing (AM), requiring simulations of laser-induced powder melting with formation and subsequent solidification of liquid metal pools, with numerous numerically challenging issues. These include modeling of liquid-solid interfaces, due to powder melting/solidification; gas-solid and gas-liquid multi-material interfaces, with representation of ambient gaseous media (air, Argon), using a fully-compressible formulation; and complex interfacial physics, including surface tension and Marangoni effects, adequate representation of fluid-solid-gas contacts and wetting phenomena.

Governing equations for melting/solidifying metallic substances are discretized with (up to) the 4th-order accurate (Reconstructed) DG method. Since we are interested in the low-speed limit, the degrees of freedom solved for are those for pressure, velocity and temperature/specific internal energy/enthalpy, while the non-linear residual vectors are formulated for conservative variables (mass, momentum and energy). The resulting set of non-linear equations is solved using the Newton-Krylov method, with the Jacobian-free version of the GMRES solver for linear iterations. For time discretization, we use a set of high-order (up to the 5th) L-stable fully-implicit discretization schemes (BDF2 and ESDIRK3-5). Due to the stiffness of the underlying physics (combination of acoustics, thermal and viscous/material strength effects), GMRES must be preconditioned. Our strategy for preconditioning is a combination of a p-multigrid method with (physics-based) operator splitting. While operator-splitting preconditioning of (finite-volume-based) Navier-Stokes solvers has been successfully deployed in the past, using a p-multigrid technique for the preconditioning of DG-based solvers is relatively new, especially in the context of the Jacobian-free Newton Krylov

(JFNK) methodology. We investigate different options of splitting governing equations for linearized residuals, rendering a reduced set of the approximate (preconditioning) Jacobian, which can be approximately solved for at the preconditioning steps of the Krylov iterations, either directly or iteratively (with Krylov- or AMG-based solvers). The figures of merit in designing successful splitting of equations and degrees of freedom (within p-multigrid) are (a) the spread/clustering of eigenvalues, (b) the condition number of the preconditioned Jacobian matrix, (c) the total number of Krylov iterations, (d) the memory imprint due to additional storage required for approximate Jacobian matrices, and (e) total CPU time per non-linear iterations. The method is tested on a classical set of benchmark problems (like lid- or thermally-driven flows), and AM-relevant natural convection problems with embedded melting/solidifying fronts.

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