Preconditioning of stochastic Galerkin method

Ivana Pultarova
Thakurova 7
Praha 6
166 29
Czech Republic
Europe
ivana@mat.fsv.cvut.cz

Stochastic Galerkin method is a popular tool for solving differential equations with uncertain data. We consider a second order scalar elliptic partial differential equation

$$-\nabla \left(a(x,y)\nabla u(x,y)\right) = b(x)$$

with Dirichlet boundary conditions. The coefficient $a(x,y)$ is not determined exactly. We use a truncated Karhunen-Loeve expansion of $a(x,y)$ and with either uniformly or normally distributed of random variables y_i. Variable x is a spatial variable. Weak formulation and finite element discretization of the space part of the solution and polynomial chaos expansion of the stochastic part of the solution lead to a system of linear equations, the dimension of which is huge. Then efficient preconditioning techniques are demanded. We introduce a hierarchical multilevel preconditioning method obtained from a splitting of approximation spaces V with respect to stochastic variables. Especially, we deal with approximation of the stochastic part of the solution by complete polynomials of order P. We consider a splitting of V into the direct sum of the complete orthogonal polynomials of order $P-1$ and of the rest of V. As one of the main results, we prove that for the uniform distribution of y_i (and thus for Legendre orthogonal polynomials), the corresponding Cauchy-Buniakowski-Schwarz constant is bounded by $\sqrt{P/(2P+1)}$ and thus the condition number of the two-by-two block preconditioned matrix of the problem is less than $3+2\sqrt{2}$ for any $P>1$.