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Discretization of Elliptic Differential Equations Using Sparse Grids and Prewavelets

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Sparse grids can be used to discretize second order elliptic differential equations on a d-dimensional cube. Using Galerkin discretization, we obtain a linear equation system with $O(N(\log N)^{d-1})$ unknowns. The corresponding discretization error is $O(N^{-1}(\log N)^{d-1})$ in the H^1-norm. A major difficulty in using this sparse grid discretization is complexity of the related stiffness matrix. Consequently, only differential equations with constant coefficients could be efficiently discretized using sparse grids for $d > 2$. To reduce the complexity of the sparse grid discretization matrix, we apply pre-wavelets. This simplifies the implementation of the multigrid Q-cylce. Furthermore, we present a new sparse grid discretization for Helmholtz equation with a variable coefficient c. This discretization utilizes a semi-orthogonality property. The convergence rate of this discretization in H^1-norm is $O(N^{-1}(\log N)^{d-1})$ for $d \leq 4$ and $O(N^{-(2/(d-2))})$ for $d \geq 5$.