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Scott MacLachlan  
**Monolithic multigrid methods for coupled multi-physics  
problems**

Department of Mathematics and Statistics  
Memorial University of Newfoundland  
St John's  
NL  
A1C 5S7  
Canada  
`smaclachlan@mun.ca`

James Adler  
Thomas Benson  
Eric Cyr  
David Emerson  
Thomas Manteuffel  
Ray Tuminaro

While block-diagonal and approximate block-factorization preconditioners are often considered for coupled problems, monolithic approaches can offer improved performance, particularly when the coupling between equations is strong. In this talk, we discuss the extension of Braess-Sarazin relaxation techniques to the linear systems resulting from the linearization and finite-element discretization of such coupled systems. Defining an easy-to-invert approximation to the (1,1) block of the system is a key step in achieving efficiency, and we demonstrate that the finite-element discretization can effectively guide this choice. Numerical results are presented for two problems, magnetohydrodynamics and flexoelectric nematic liquid crystals. In both cases, geometric multigrid with Braess-Sarazin relaxation shows excellent performance and scaling with problem size and physical parameters.