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Robust parallel hierarchical hybrid multigrid methods for the Stokes equations

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In this talk we present robust multigrid methods for the Stokes equations on hierarchical hybrid grids (HHG). The special design of the method, i.e., a compromise of structured and unstructured grids, fits the flexibility of finite elements and the efficiency of geometric multigrid methods. It provides excellent scalability up to a million parallel threads and can solve in excess of 10^{12} unknowns in less than 2 minutes computing time on state of-the-art supercomputers. In particular, three types of solvers are presented and compared to each other with respect to the time to solution as well as operator applications. Namely, a Schur complement CG and the Krylov subspace method MINRES which is preconditioned by a block diagonal preconditioner, consisting of a parallel multigrid and a lumped mass matrix. Further, we investigate so-called all at once solution techniques. In this particular case Uzawa-type smoothers have been found to be an attractive choice, since they are numerically cheap and can be implemented with only nearest-neighbour communication. An important observation is the number of operator applications within the solution process. Here we discover that the numbers for the Schur complement CG and the MINRES with block diagonal preconditioning are similar, while the UzawaMG profoundly profits from the comparatively fewer applications. These numbers are also reflected in the time to solution for the individual solvers. Several numerical examples illustrate the performance of the presented methods. Moreover, an application to Earth mantle convection problems will be given.