LIU Hao

Local Defect Correction multigrid Method coupled with the Zienkiewicz-Zhu *a posteriori* error estimator in elastostatics Solid Mechanics

CEA/Cadarache
DEC/SESC/LSC
Building 151
13108 SAINT-PAUL-LEZ-DURANCE CEDEX
France
hao.liu@cea.fr
Ramière Isabelle
Lebon Frédéric

Local multigrid methods provide a technique to solve problems which locally require high accuracy (cracks, discontinuous boundary conditions, entrant corners etc.) in acceptable computational times and memory space. These methods have several advantages such as fast resolution on each sub-grid, work on local fine meshes only...

We propose here to use the Local Defect Correction (LDC) multigrid method, proposed by Hackbush [3], to solve Solid Mechanics Problems. From an initial coarse mesh, this method focuses on recursively adding local sub-grids in areas where higher accuracy is required. We keep adding levels of refinement until the finest grid has the desired accuracy. Then prolongation and restriction operators are used to link the different level of grids. It should be noted that this method is generic : solver, refinement ratio , mesh type, model etc. could be different (or not) between the several levels of refinement.

In order to automatically detect the zones to be refined, we propose to couple the LDC multigrid method with the Zienkiewicz-Zhu *a posteriori* error estimator [5]. Based on the fact that the stress field calculated by the finite element method is discontinuous between elements, the Zienkiewicz-Zhu *a posteriori* error estimator constructs a smoothed stress field. Then the error estimator is evaluated by the difference between the finite element stress solution and the smoothed stress fields. A strategy of coupling the LDC multigrid method with the Zienkiewicz-Zhu *a posteriori* error was already introduced in [1, 2]. The authors proposed to refine the elements for which the indicator is greater than $\alpha\%$ of the maximum error. This strategy is easy to implement but requires the knowledge of the α (which may depend of the problem) and the maximal number of sub-grids (because this indicator never stops). We propose here an automatic procedure of refinement working directly on the elements for which the stress error is superior than a threshold (relative error) given by the user.

In this way, we avoid the dependence on a arbitrary coefficient.

We apply this method on an industrial test case that is simulation of the Pellet-Cladding mechanical Interaction (PCI) in Pressurized Water Reactors (PWR) [4]. Precise simulations of this problem require cells of 1 μm for a structure of 1 cm. The LDC multigrid method could be efficient to overcome this issue. We present results obtained on the 2D linear elasticity test cases with discontinuous boundary condition. Whatever the thresholds set by user, the proposed coupled method gives really satisfactory results. Memory space and CPU time saving will be also pointed out.

Bibliography

[1] L. Barbié, I. Ramière, and F. Lebon. Strategies around the local defect correction multi-level refinement method for three-dimensional linear elastic problems. *Computers and Structures*, 130:73–90, 2014.

[2] L. Barbié, I. Ramière, F. Lebon, and J. Sercombe. AMR methods in solids mechanics for pellet-cladding interaction modelling. In D. Aubry, P. Díez, B. Tie, and N. Parés, editors, *V International Conference on Adaptive Modeling and Simulation*, pages 70–81, 2011. 6-8 June, Paris, France.

[3] W. Hackbusch. Local Defect Correction Method and Domain Decomposition Techniques. *Computing Suppl. Springer-Verlag*, 5:89–113, 1984.

[4] B. Michel, C. Nonon, J. Sercombe, F. Michel, and V. Marelle. Simulation of pellet-cladding interaction with the pleiades fuel performance software environment. *Nuclear Technology*, 182, 2013.

[5] O.C. Zienkiewicz and J.Z. Zhu. A simple error estimator and adaptive procedure for practical engineering analysis. *International Journal for Numerical Methods in Engineering*, 24:337–357, 1987.