Andrei Draganescu

Optimal Order Multigrid Preconditioners for Linear Systems Arising in the Semismooth Newton Method Solution Process of a Class of Control-Constrained Problems

University of Maryland
Baltimore County
Department of Mathematics and Statistics
1000 Hilltop Circle
Baltimore
MD 21250
draga@umbc.edu
Jyoti Saraswat

In this work we present a new multigrid preconditioner for the linear systems arising in the semismooth Newton method solution process of certain control-constrained, quadratic distributed optimal control problems. Using a piecewise constant discretization of the control space, each semismooth Newton iteration essentially requires inverting a principal submatrix of the matrix entering the normal equations of the associated unconstrained optimal control problem, the rows (and columns) of the submatrix representing the constraints deemed inactive at the current iteration. Previously developed multigrid preconditioners by Drăgănescu [*Optim. Methods Softw.*, 29 (2004), pp. 786–818] for the aforementioned sub matrices were based on constructing a sequence of conforming coarser spaces, and proved to be of suboptimal quality for the class of problems considered. Instead, the multigrid preconditioner introduced in this work uses non-conforming coarse spaces, and it is shown that, under reasonable geometric assumptions on the constraints that are deemed inactive, the preconditioner approximates the inverse of the desired submatrix to optimal order. The preconditioner is tested numerically on a classical elliptic-constrained optimal control problem and further on a constrained image-deblurring problem.